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ARTICLE

Towards Quantitatively Understanding the Complexity of Social-Ecological Systems—From Connection to Consilience

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Abstract The complexity of social-ecological systems (SES) is rooted in the outcomes of node activities connected by network topology. Thus far, in network dynamics research, the connectivity degree (CND), indicating how many nodes are connected to a given node, has been the dominant concept. However, connectivity focuses only on network topology, neglecting the crucial relation to node activities, and thereby leaving system outcomes largely unexplained. Inspired by the phenomenon of "consensus of wills and coordination of activities" often observed in disaster risk management, we propose a new concept of network characteristic, the consilience degree (CSD), aiming to measure the way in which network topology and node activities together contribute to system outcomes. The CSD captures the fact that nodes may assume different states that make their activities more or less compatible. Connecting two nodes with in/compatible states will lead to outcomes that are un/desirable from the perspective of the SES in question. We mathematically prove that the CSD is a generalized CND, and the CND is a special case of CSD. As a general, fundamental concept, the CSD can facilitate the development of a new framework of network properties, models, and theories that allows us to understand patterns of network behavior that cannot be explained in terms of connectivity alone. We further demonstrate that a co-evolutionary mechanism can naturally improve the CSD. Given the generality of co-evolution in SES, we argue that the CSD is an inherent attribute rather than an artificial concept, which underpins the fundamental importance of the CSD to the study of SES.

Keywords Complex networks · Consilience degree · Co-evolution · Disaster risk reduction · Social-ecological systems

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1 Introduction

In many natural and social-ecological systems (SES), the physical topology of networks is only part of what determines their performance (Ostrom 2009; Ball 2012). Much also depends on the function of the individual nodes and the ways in which the nodes interact with each other. In engineering systems, the rule "$1 + 1 = 2$" often applies, implying that the connection and its topology are the focus. The concept of "degree of connectedness" (CND)

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precisely reflects this, and has promoted unprecedented advances in system science in the last two decades (Albert and Barabási 2002; Boccaletti et al. 2006). For instance, one of the most important findings in system science is that the CND distribution of most real-world complex networks, such as the World Wide Web (WWW) (Huberman and Adamic 1999), airline networks (Burghouwt et al. 2003), and phonecall networks (Aiello et al. 2000), significantly deviates from a Poisson distribution, but has a power-law tail or a scale-free property (Barabási and Albert 1999). However, in SES, the effects of "$1 + 1 > 2$" and "$1 + 1 < 2$" are also observed, which suggests that which two nodes are connected may be more important than the connection itself, and therefore our research focus may need to be shifted from structural connectedness to functional integration. Although the weight of connections may partially help to describe such effects (Albert and Barabási 2002; Boccaletti et al. 2006), node activities are often the main cause (Peyton Young 1998; Daido and Nakanishi 2004), but are largely ignored. As a result, how to maximize the effect of "$1 + 1 > 2$" and to minimize the effect of "$1 + 1 < 2$" is beyond the scope of CND-based network approaches. As clearly pointed out in many studies of SES, despite great potential, existing topology-focused CND network theories need innovative improvements before they can become effective methods to address the complexity of SES (OECD 2011; Ball 2012; Helbing 2013). For example, a widely acknowledged aspect of CND research is that high-degree nodes are more important than low-degree nodes in terms of structural robustness against intentional perturbations (Callaway et al. 2000; Cohen et al. 2001). However, a recent study shows that, once one takes node activities into account to assess the dynamical robustness of a system, low-degree nodes are actually more important than high-degree nodes in the face of intentional perturbations (Tanaka et al. 2012). Figure 1 gives an example from daily life on how to properly network people according to their expertise in order to achieve optimum management performance. In this example, CND-based network theories can hardly distinguish the two systems, but if expertise similarity in a sub-team will lead to good performance, then we know that team 1 is better than team 2. Despite the theoretical success of the CND in studying network structure, most realistic case studies of network systems have to consider both topology and node activities simultaneously. Examples are neural networks (Daido and Nakanishi 2004), power grids (Blaabjerg et al. 2006), epidemic dynamics (Pastor-Satorras and Vespignani 2001), cascading effects in disaster spreading (Helbing 2013), individual fitness (Caldarelli et al. 2002), social norms and collaborative expectations (Peyton Young 1998), co-evolutionary dynamics (Nardini et al. 2008; Aoki and Aoyagi 2012), and data mining (Hric et al. 2016; Peel et al. 2017). However, in these studies the definitions of node activities and the methods to analyze them are highly problem-specific and have a dynamic nature. There is no general method to study the functional fusion of topology and node activities in a static network.

In real-world network systems, the macrosystem output reflects the collective performance of all micronode activities, and to contribute to such a collective performance, each node, through network topology, not only supports its neighboring node activities, but also integrates neighboring node resources to enhance its own activity (Ball 2012). For example, "consensus of wills and coordination of activities" between individuals plays a crucial role in a social system if it is to achieve good performance in disaster risk management (Hu et al. 2014; Shi et al. 2014; Bodin and Nohrstedt 2016). Inspired by such observations, this article proposes a general, fundamental network property concept, named "degree of consilience" (CSD). The term "consilience" literally refers to the principle that evidence from independent, unrelated sources can "converge" to a strong conclusion or a scientific consensus (Wilson 1999). In sustainability science, consilience is particularly used to highlight the importance of a massive global cooperative effort and integrated cross-disciplinary coordination (Lee and all members of Editorial Board 2009; Wilson 2009). The proposed network property CSD here, by adopting the term "consilience," attempts to evaluate the collective contribution of all factors (topology and node activities) in a networked system towards its performance in terms of certain functional goals. In particular, the concept of the CSD may provide a new methodological tool for the research on global environmental change because in such research the integration of knowledge from different disciplines, collective action, and public support are of paramount importance (Alexander et al. 2015; Bernauer et al. 2016; Cox et al. 2016).

2 The Concept of Consilience Degree (CSD)

Suppose there is a networked system, whose topology is given by $G(V, E)$, composed of node set V and edge set E–V has N_N nodes and E has N_E edges. Let the adjacency matrix record all edges, that is, $M_A(i, j) = 1$ means that there is an edge between nodes i and j, and otherwise $M_A(i, j) = 0$. The degree of connectedness (CND) of node i, indicating how many other nodes are connected to node i, is mathematically defined as:

$$k_{CN,i} = \sum_{j=1}^{N_N} M_A(i,j). \tag{1}$$

The consilience degree (CSD) of node i in this study is defined as:

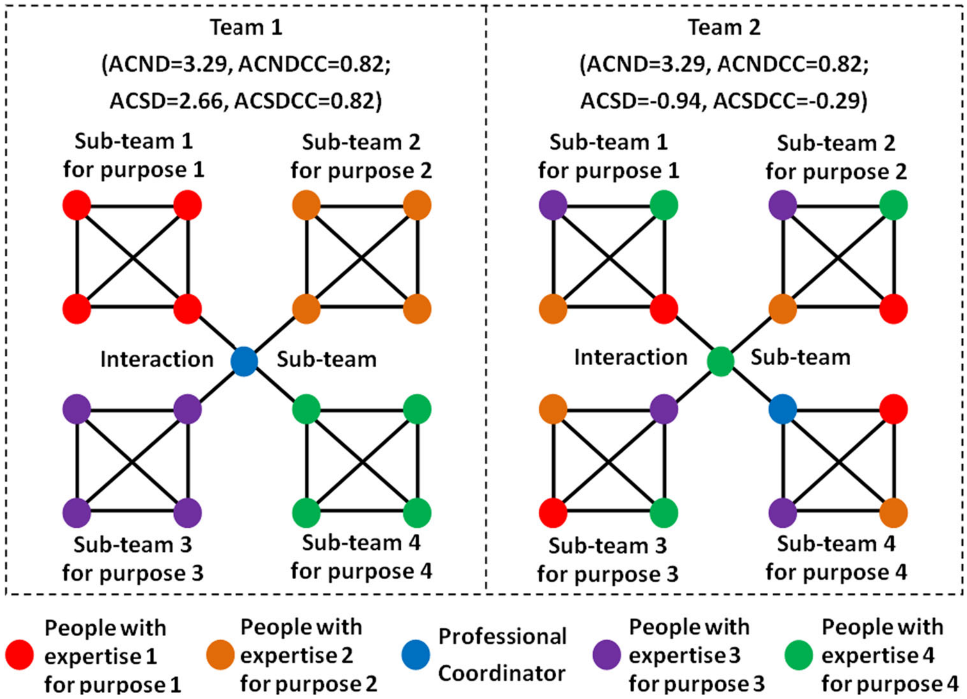


Fig. 1 Networks of collaborating people. Despite the fact that they have exactly the same network topology and human resources, team 1, by organizing itself according to the similarity in expertise of its members, is likely to achieve a better performance than team 2, given that expertise similarity that matches the purpose in a sub-team will lead to good performance. Connection degree (CND) based network theories largely fail to quantify or distinguish the teams' capability to serve their purpose, because the CND-based network properties of these two teams are exactly the same—for example, they have the same average CND (ACND) and the same average CND-based clustering coefficient (ACNDCC). However, if one brings the functional expertise of the members of the team into play, which can be measured in terms of differences in knowledge, skills, and style between different experts, the average consilience degree (ACSD), as well as the average CSD-based clustering coefficient (ACSDCC), can capture and describe the overall difference in performance of the two teams, revealing that team 1 will perform better because it has a larger ACSD. On how to calculate ACND, ACNDCC, ACSD, and ACSDCC, see Sects. 2 and 3 for details

$$k_{CS,i} = \sum_{j=1}^{N_N} M_A(i,j) \times f_{CS}(\theta_i, \theta_j), \tag{2}$$

where $\theta_i = [\theta_{i,1}, \ldots, \theta_{i,N_{ASD}}]$ represents the activity state of node i, and $N_{\mathrm{ASD}} \geq 1$ the dimension of that activity state (in many natural, engineering, or social-ecological systems, nodes have multi-dimensional activity states); $\underline{f_{CS}} \leq f_{CS}(\theta_i, \theta_j) \leq \bar{f}_{CS}$ is called the "consilience function," determining how the states of nodes i and j will affect the overall performance if the nodes are connected, and $\underline{f_{CS}}$ and $\bar{f}_{CS}$ are the lower and upper bounds, respectively; $f_{CS}(\theta_i, \theta_j)$ may be of any form depending on the nature of the system concerned. In Eq. 2, $M_{\mathrm{A}}(i, j)$ represents the network topology, and $f_{CS}(\theta_i, \theta_j)$ introduces the node activities that are the focus of this study. For the sake of simplicity, we assume $f_{CS}(\theta_i, \theta_j) = \cos(\theta_i - \theta_j)$ in all simulations of this article.

In the real world, individual nodes may act differently, but their activities need to serve the same systemic goal. Through the network topology, nodes interact with each other. When a specific systemic goal is concerned, due to the differences in node activities, some nodes, if connected, may interact well, while some others, if connected, will conflict with each other. Many factors, such as signal synchronization, compatibility of facilities, complementarity or similarity of expertise (for example, Fig. 1), willingness of collaboration, social opinion, personal attitude, and cultural (dis)similarity usually play a role at least as crucial as that of physical connections in determining the performance of the connected nodes. In general, the node activity state and the consilience function in Eq. 2 can correctly describe such real-world situations. For example, if the similarity in node activities helps performance, then we can define $f_{CS}(\theta_i, \theta_j) = 1$ when $\theta_i = \theta_j$, and if complementarity between node activities is desirable, then we may have $f_{CS}(\theta_i, \theta_j) = 1$ when $|\theta_i - \theta_j| \geq \theta_T$, where θ_T is a problem-specific threshold.

Given $-1 \leq f_{CS}(\theta_i, \theta_j) \leq 1$, it follows that $-k_{CN,i} \leq k_{CS,i} \leq k_{CN,i}$. In the case where $f_{CS}(\theta_i, \theta_j) = 1$ for any pair of connected nodes in the system, CSD becomes exactly CND, that is, $k_{CS,i} = k_{CN,i}$. From Eqs. 1 and 2, one may conclude that CSD is an extension of CND, while CND is just a special case of CSD. Therefore, CSD is a more general, more fundamental network property than CND. Basically, if a node connects to other nodes that have more states compatible to its own, then the node has a higher CSD (for example, see Fig. 2), which may indicate that the node has a better capability of integrating available resources in the system. Such a capability is fundamental for the system if it is to achieve a certain systemic goal, but traditional network properties, such as CND, synchronization, clustering coefficient, and robustness, can hardly capture or measure that capability. In real-world network systems such as SES, there is often a "being together—but better not" situation (for example, team 2 in Fig. 1). CND studies only the first part, the "being together," while CSD completes the picture by disclosing the second part "but better not."

According to the definition of Eq. 2, an isolated node i has a CSD value of 0, which complies with common sense. Even for a node with $k_{CN,i} > 0$, it could still have $k_{CS,i} = 0$ if the connected nodes are equally conflictive to each other, which also makes sense in real-world systems. For example, a machine needs two external accessories to function properly, but it is connected to two accessories that are completely incompatible with each other due to different makers. Therefore, the machine can be viewed as having been connected to nothing. Another example is, if one needs advice from two equally trustworthy friends, but whose advice is completely contradictory. In this situation it makes no difference if no friends at all are consulted. Therefore, CSD is a network property that CND fails to capture. Please note that, as demonstrated in Fig. 1, we usually use average CND (ACND) and average CSD (ACSD) to study the performance of a network system, and ACND and ACSD, denoted as $\overline{k}_{CN}$ and $\overline{k}_{CS}$, respectively, are calculated as follows:

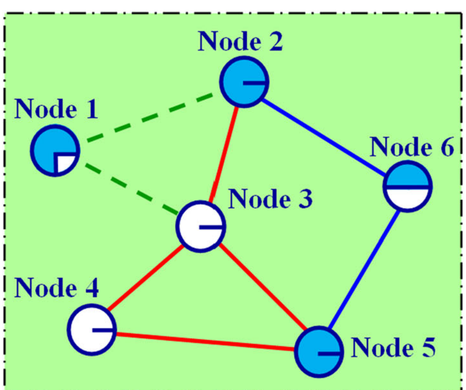
$$\overline{k}_{CN} = \frac{1}{N_N}\sum_{i=1}^{N_N} k_{CN,i} \tag{3}$$

$$\overline{k}_{CS} = \frac{1}{N_N}\sum_{i=1}^{N_N} k_{CS,i}. \tag{4}$$

Attention should also be paid not to confuse CSD with network synchronization, which can be assessed by the average activity state difference

$$\overline{\Delta\theta} = \frac{1}{N_N(N_N-1)}\sum_{i=1}^{N_N}\sum_{j=1}^{N_N}\left|\theta_i - \theta_j\right|. \tag{5}$$

Roughly speaking, a network system with a smaller $\overline{\Delta\theta}$ might often have a larger average consilience degree, that is, ACSD $\overline{k}_{CS}$ as defined in Eq. 4. However, depending on network topology, it does happen that (1) two network systems with the same $\overline{\Delta\theta}$ may have different $\overline{k}_{CS}$ values, and (2) a network system with a larger $\overline{\Delta\theta}$ may have a larger $\overline{k}_{CS}$ than a system with a smaller $\overline{\Delta\theta}$, even though the consilience function $f_{CS}(\theta_i, \theta_j)$ is assumed to favor similar activity states. Therefore, consilience degree is also a property different from synchronization.



Node i	1	2	3	4	5	6
Node activity state θ_i	1.5π	2π	0	0	2π	π
Connection degree (CND) $k_{\mathrm{CN},i}$	2	3	4	2	3	2
Consilience degree (CSD) $k_{\mathrm{CS},i}$	0	0	3	2	1	-2
CND clustering coefficient	1	0.33	0.33	1	0.33	0
CSD clustering coefficient	1	0	0.16	1	0.33	0
Neighbor CSD coefficient	0	0	0.75	1	0.33	-1
Global CSD coefficient	0	0	0.6	0.4	0.2	-0.4

Connection with positive effect — Connection with negative effect — Connection with no effect

Fig. 2 Differences between some connection degree (CND) based and consilience degree (CSD) based network properties. Between a pair of connected nodes, because of the difference in their activity states, there may be a positive effect (indicated as a red line), no actual effect (green dashed line), or even a negative effect (blue line). The values of CND and CSD are rather independent of each other. A large CND does not necessarily mean a large CSD (for example, see node 2 and node 4). CND can therefore not replace CSD. The CND-based clustering coefficient shows that the cluster of nodes 1, 2, and 3 is exactly the same as the cluster of nodes 3, 4 and 5. However, the CSD-based clustering coefficient tells us that nodes 3, 4, and 5 form a stronger cluster, which better fits with reality when factors such as collaborative attitude are considered. Therefore, the CSD version of the traditional network properties provides a better understanding of real-world network systems. Newly created network properties purely based on CSD further enrich the application potential of the concept of CSD. For example, to fairly compare node capability of integrating neighbor or system resources, regardless of how many neighbors a node has or what network scale the system has, a neighborhood or global CSD coefficient needs to be used rather than CSD itself

A clustering coefficient describes how tense a node and its neighbors are connected to each other by edges, and for node i it is usually calculated as

$$c_{CC,i} = \frac{2n_{E,i}}{k_{CN,i}(k_{CN,i} - 1)}, \tag{6}$$

where $n_{E,i}$ is the number of all edges existing in the cluster, which is composed of node i and all its $k_{\mathrm{CN},i}$ neighbors. As shown in Eq. 6, the clustering coefficient is defined purely based on CND. The average CND-based clustering coefficient (ACNDCC) is often used to study network systems (Albert and Barabási 2002; Boccaletti et al. 2006), and ACNDCC, denoted as $\bar{c}_{CC}$, is calculated as

$$\bar{c}_{CC} = \frac{1}{N_N}\sum_{i=1}^{N_N} c_{CC,i}. \tag{7}$$

Cluster is an important concept from the reality point of view, because it is often observed that individuals, represented by nodes, with similar features, measured by activity states, will cluster in a network system. However, the property of the clustering coefficient only discloses part of the picture, as Eq. 6 has nothing to do with node activity states. This means a cluster of conflictive nodes may still have a high clustering coefficient, which is somehow against common sense. In such a case, CSD may serve as a much less confusing index: no matter how many edges exist in a cluster, as long as those nodes are conflictive to each other, node i will have a small CSD value, which may indicate it is a weak cluster.

Robustness/vulnerability is another very important network property, and a scale-free network is vulnerable to intended attacks to hub nodes (Albert and Barabási 2002; Boccaletti et al. 2006). Given two hub nodes with the same CND, then do they also have the same vulnerability to intended attack? According to the traditional definition of robustness, removing either of the two hub nodes will lead to the same network degradation. However, the reality may tell a different story. Imagine two managers, who are each responsible for the same number of employees. In one group, all employees are highly supportive of the manager, while in the other group, everyone fights against each other. Which manager is likely to fail in his/her career? As a more general question, given two network systems—one has a scale-free topology with hubs well connected to nodes of similar activity states, and the other is randomly structured regardless of the node activity state distribution—which system will be more likely to collapse when facing intended attacks? Taking CSD into account, a scale-free network could turn out to be more robust than a random network in the face of intended attacks.

3 A New Theoretical Framework Based on CSD

The CND concept has developed into a theoretical framework that is composed of many network properties, models, and theories, and is of great use for studying network structure. Similarly, the CSD concept can be modified and extended to create a new theoretical framework that will enable the study of the functional fusion of network topology and node activities and can significantly widen and deepen our understanding of complex network systems.

3.1 New Network Properties

The CSD given by Eq. 2 is a very basic definition and can be modified and/or extended. We propose a modified but still general definition: the neighborhood consilience coefficient (NCSC). For node i, its NCSC is calculated as

$$c_{NCSC,i} = \frac{k_{CS,i}}{k_{CN,i}}. \tag{8}$$

According to Eq. 2, $k_{CS,i}$ can be any real number, while $c_{NCSC,i}$ in Eq. 8 is always within the range $[-1, 1]$. Therefore, $c_{NCSC,i}$ can be viewed as a normalized $k_{CS,i}$; NCSC can be used to assess how efficient a node integrates its neighbor resources. For example in Fig. 2, node 3 has 4 neighbors and $k_{CS,3} = 3$, and node 4 has 2 neighbors and $k_{CS,4} = 2$. Although $k_{CS,3} > k_{CS,4}$, node 4 is actually more efficient than node 3 in terms of integrating neighbor resources, because, according to Eq. 8, $c_{NCSC,4} = 1 > c_{NCSC,3} = 0.75$.

In a network system, no matter whether two nodes are connected or not, they can be viewed as available resources to each other, because when optimizing the system, one may add an edge between the two nodes if necessary. Therefore, we often need to consider how well a node integrates all available resources in the system rather than its neighbor resources. To this end, we introduce another modified but also general definition: the global consilience coefficient (GCSC), which is calculated for node i as

$$c_{GCSC,i} = \frac{k_{CS,i}}{N_N - 1}. \tag{9}$$

In theory, GCSC is within the range $[-1, 1]$, but for node i with node degree $k_{\mathrm{CN},i}$, the maximum value for $c_{GCSC,i}$ is $\frac{k_{CN,i}}{N_N - 1}$. To give a real-world example, suppose a political party is preparing for a presidential election. The chance for the party leader to become the president is determined not only by how well all party members are involved (measured by NCSC), but also by how well the public are contacted and convinced (indicated by GCSC). Moreover, GCSC is very useful for fairly comparing network systems with different scales, that is, N_N values, which is almost a mission impossible for NCSC.

Some more sophisticated or problem-specific modifications can be introduced to the definition of CSD in Eq. 2. For example, activity state may not be enough to describe the difference in node activities, and activity amplitude is often also needed. Assuming the activity amplitude of node i is $a_i > 0$, we can redefine CSD as

$$k_{CS,i} = \sum_{j=1}^{N_N} a_j \times M_A(i,j) \times f_{CS}(\theta_i, \theta_j). \tag{10}$$

In some systems, edges may have different connecting effects, which can be assessed by a weight on the edge. Given the edge connecting node i and node j has a weight $w_{i,j}$, then the CSD in Eq. 10 can be further modified as

$$k_{CS,i} = \sum_{j=1}^{N_N} w_{i,j} \times a_j \times M_A(i,j) \times f_{CS}(\theta_i, \theta_j). \tag{11}$$

The definitions of NCSC in Eq. 8 and GCSC in Eq. 9 can be modified accordingly. For example, if both node activity amplitude and edge weight need to be considered, then we may define

$$c_{NCSC,i} = \frac{1}{k_{CN,i} \max\limits_{j=1,\ldots,k_{CN,i}} (w_{i,j} \times a_j)} \sum_{j=1}^{N_N} w_{i,j} \times a_j \times M_A(i,j) \times f_{CS}(\theta_i, \theta_j), \tag{12}$$

$$c_{GCSC,i} = \frac{1}{(N_N - 1) \max\limits_{k,j=1,\ldots,N_N} (w_{k,j}) \max\limits_{j=1,\ldots,N_N} (a_j)} \times \sum_{j=1}^{N_N} w_{i,j} \times a_j \times M_A(i,j) \times f_{CS}(\theta_i, \theta_j). \tag{13}$$

For whichever definition, the average value based on all nodes in a network system may then be used to assess the overall network consilience.

As discussed in Sect. 2, CND is a special case of CSD. Since many traditional network properties—for example, clustering coefficient and assortativity—are developed largely based on CND, we may then define CSD-based versions of such network properties. For example, for node i, we may recalculate the clustering coefficient based on the concept of CSD

$$c_{CSCC,i} = \frac{\sum_{k,j \in \Omega_{N,i}, k \neq j} M_A(k,j) \times f_{CS}(\theta_k, \theta_j)}{k_{CN,i}(k_{CN,i} - 1)}, \tag{14}$$

and the average CSD-based clustering coefficient (ACSDCC) is

$$\bar{c}_{CSCC} = \frac{1}{N_N} \sum_{i=1}^{N_N} c_{CSCC,i}, \tag{15}$$

where $\Omega_{N,i}$ denotes the set of neighbor nodes of node i. For a cluster of nodes that have many edges between each other but observe rather conflictive node activity states, one will get a large traditional CND-based clustering coefficient according to Eq. 6, but a small and even negative CSD-based clustering coefficient according to Eq. 14, as illustrated in Figs. 1 and 2. For example, in the case of team 2 of Fig. 1, the CND-based clustering coefficient (average value 0.82) gives a misleading impression that every corner sub-team is well connected, but according to the CSD-based clustering coefficient (average value − 0.29), all sub-teams in team 2 are badly organized, given that expertise similarity positively impacts on performance. This proves that the concept of CSD opens another door for us to understand network systems.

3.2 New Network Models

Similar to the fact that many traditional network properties are defined based on CND, many existing network models are developed mainly by referring to the concept of CND. For example, as one of the most acknowledged network models, the preferential attachment model uses the CND of a node to determine the probability of whether to add a new edge to that node (Barabási and Albert 1999). Basically, a new edge will more likely link to a node with a larger CND. Obviously, it is not difficult to apply the preferential attachment mechanism to simulate CSD-oriented network systems. All we need to do is to simply replace the probability calculation part in the model of Barabási and Albert (1999), in order to make a node with a larger CSD to have a larger probability of being connected. Then, the new network model, CSD-preferential, will not only generate scale-free topologies, but also achieve a good overall network consilience, which will be demonstrated by the simulation results in Sect. 4.

Does a system with a good network consilience always have a scale-free topology? To answer this question, we need to develop another CSD-oriented network model, but without the preferential attachment mechanism in Barabási and Albert (1999). In the new model, each time (1) two unconnected nodes are randomly selected, and (2) the probability of adding a new edge between them depends on the difference in their activity states. Basically, a smaller difference in activity states means a larger probability of connection. One may use the following function to calculate the activity -state -difference-based probability

$$p_C(i,j) = \frac{(\alpha + 1 + f_{CS}(\theta_i, \theta_j))^{\beta}}{\sum_{k=1}^{N_N} \sum_{h=k+1}^{N_N} (\alpha + 1 + f_{CS}(\theta_k, \theta_h))^{\beta}}, \tag{16}$$

where $\alpha > 0$ makes sure that even the two most conflictive nodes may have a chance to be connected, and $\beta > 0$ determines how strong the influence of activity state difference is on the probability. As will be shown in the simulation results, this new model can achieve good network consilience, but does not necessarily require a scale-free topology. Therefore, as emphasized throughout this article, topology is just one part of network systems. Once node activities cannot be ignored, pure topology-based analyses could become less useful or even misleading.

3.3 New Network Optimization Considerations

The concept of CSD also demands new considerations for network optimization problems. Given N_N nodes with various preset activity states, due to limited resources, we can only establish N_E edges between these nodes. Then, how to allocate N_E edges in order to achieve the maximum average consilience degree (ACSD)? This optimization problem makes no sense in terms of CND, because no matter how N_E edges are allocated, the average connection degree (ACND) remains the same as $2N_E/N_N$. Differently, the optimization of edge allocation is extremely important in terms of CSD, and it also has a broad real-world application background. For example, when a social-ecological system is facing environmental pressure, how to organize various stakeholders according to their interests and expertise is a challenging task (Adger 2006; Young 2010), and the optimization of CSD may reveal some helpful clues.

We first propose a theoretical network model to generate a system with the theoretically maximal ACSD. In this model, suppose there is a central governor who is responsible for allocating every single edge according to the global optimality. Basically, when the lth edge is to be allocated, $l = 1,\ldots,N_E$, there are $((N_N - 1)N_N/2 - l+1)$ options, and each option is associated with two nodes, say node i and node j. Then, the option with the maximal $f_{CS}(\theta_i, \theta_j)$ value among all these $((N_N - 1)N_N/2 - l+1)$ options will be chosen to allocate the lth edge. In this way, the theoretically maximal ACSD can be achieved.

However, many real-world network systems often lack such a central governor, and individual nodes have the right to decide where to set up their own edges. Such networks are decentralized self-organizing systems, and all nodes take the initiative to compete for edge resources. To optimize their CSD, we have another theoretical network model, where a node, once it receives the resource of a new edge, will set up a new edge in such a way that the node maximizes its own CSD. In this model, every time when a new edge is to be set up, a node needs to be chosen randomly. Assuming node i with $k_{CN,i} < (N_N - 1)$ is chosen, then there are $(N_N - 1 - k_{CN,i})$ options for node i to set up the new edge. The option with the maximal $f_{CS}(\theta_i, \theta_j)$ value among all these $(N_N - 1 - k_{CN,i})$ options will be chosen to set up the new edge. This model cannot guarantee the global optimality in terms of CSD, but it may better fit in the reality, such as in a social-ecological system, where various stakeholders often have the full control of their own decisions, and when choosing collaborative partners, they usually pursue the maximization of their own interests.

The optimization of CSD can be extended to cover more considerations. For instance, besides the $f_{CS}(\theta_i, \theta_j)$ value, the distance between two nodes may also influence the decision of allocating a new edge. Usually, a larger distance between two nodes may result in a bigger cost for setting up the edge and a lower connection efficiency. There is an old Chinese saying "Water far away is of no use to a thirsty man." Even though two nodes have supportive activity states, due to a long distance, the supporting effect between the nodes may be largely weakened. Therefore, we need to modify consilience optimization models by taking into account the influence of distance. A simple illustration of distance-related modification will be given in the simulation results of Sect. 4, but in general, the modification may differ largely depending on specific concerned systems.

3.4 Potential of Applying CSD to Study Dynamic Network Systems

It should be noted that the node activity state is treated here as a rather general static concept, and it is not necessarily related to any particular network dynamics such as a coupling function, a limit-cycle oscillation, or time-varying behavior, although it can be. Therefore, the concept of the consilience degree (CSD) is basically also a static network property, in the same way the connection degree (CND) is a static network property. However, the static nature of CSD does not mean it cannot be applied to studying dynamic network systems. Actually, the CSD exhibits great potential for the study of dynamic network systems, and there are at least three ways to apply the CSD to such systems.

First, a dynamic network system can be discretized into a series of static network systems at different time instants, which is the way how dynamic systems are treated in research. At each time instant, we can take a snapshot of the dynamic network system, and such a snapshot constitutes a static network system. Therefore, CSD as well as all

CSD-based properties and models can be used to study the static snapshot of a dynamic network system. For a static network system, CSD can be used to generally describe the degree to which diversified node activities in the system are supportive of each other. For a dynamic network system, CSD can be calculated at each time instant, just like other system dynamical properties, and then the dynamic change in CSD can be used to study why it changes and how its change contributes to the system dynamics/evolution.

Second, in a dynamic network system, each node usually has its own dynamic activity/function, which determines the change of node activity state and is largely influenced by the interplay between nodes. How well a node is functioning in terms of a specific systemic goal may largely depend on how supportive or disturbing its neighboring node functions are. The concept of CSD is a key factor in describing such a dynamic activity/function of nodes. For instance, when simulating the performance of a system against external attacks, we often need to consider the recovery speed of nodes after attacks, that is, the time it takes a node to recover from an attack. In such a dynamic network system, if a node can quickly recover from a previous attack, then it will stand a better chance to survive a series of attacks. In general, the recovery speed of a node depends on not only the features of the node, but also the supportive/disturbing effects of its neighboring nodes. For example, after a natural hazard-induced disaster, whether impacted community members will help or loot each other is a key factor that will largely determine whether the community can soon thrive again or not. So CSD is an inherent part of the dynamics of such network systems.

In a more general case of dynamic network systems, both node activity states and connections between nodes may change over time. For instance, in many natural and social-ecological systems, both node activity states and network topology keep changing because of self-organizing, self-adapting, and/or co-evolutionary dynamics. In such a system, each node may change its activity state and connections from time to time by learning from and adapting to its dynamic environment. Consilience theory can help to understand/find a proper and even the best way of achieving healthy, sustainable system dynamics. For example, in coping with global climate change, multiple stakeholders in co-evolutionary social-ecological systems keep changing their attitudes and behaviors, in particular interactions/relationships between each other. What kind of policies and/or regulations might promote/prevent beneficial/harmful changes in their attitudes and behaviors over time is a potential application area of consilience theory. As will be illustrated in Sect. 4, the CSD concept has great potential for studying such co-evolutionary systems.

It should be noted that the study of a dynamic network system is usually highly problem specific, because the dynamics may differ significantly in different systems. In Sect. 4, we will design a co-evolutionary network model where both node activity states and connections between nodes co-evolve under CSD-based rules inspired by the selfish and following-others behaviors of individuals in real-world systems.

4 Simulation Results

In this section, we present some simulation results to demonstrate the importance and potentials of CSD in terms of both theoretical and application research. The simulation results have two parts. One part aims to reveal the differences between CND-based and CSD-based network properties and models. The other part reports a CSD-based model simulating co-evolutionary mechanisms in order to prove that for co-evolutionary network systems, CSD is an inherent attribute rather than an artificial concept.

4.1 Comparative Results between CSD Theory and CND Theory

Eight models are used to generate network topologies: six are based on consilience degree (CSD), and two are based on connection degree (CND). The model based on Eq. 16 sets the connecting probability according to the conflictive situation of node activity states, and is denoted as CSDPD. The other CSD-based model employs a CSD preferential attachment mechanism, and is denoted as CSDPA. For comparative purposes, two CND-based models are also used, one is the random connection model of Watts and Strogatz (1998), denoted as CNDRC, and the other is the scale-free model of Barabási and Albert (1999), denoted as CNDPA. In the simulation, node activity state is randomly generated within the range of $[0\ 2\pi]$, and $f_{CD}(\theta_i, \theta_j)$ is set as $\cos(\theta_i - \theta_j)$. Unless specified otherwise, for CNDRC, the random connection probability is 0.15, for CSDPA and CNDPA, the preferential attachment probability is formulated as

$$p_{CSDPA}(i,j) = \frac{\alpha + (2 + f_{CS}(\theta_i, \theta_j)(1 + c_{NCSC,i}))^{\beta}}{\sum_{k=1,\ldots,N_N,k\neq j} (\alpha + (2 + f_{CS}(\theta_k, \theta_j)(1 + c_{NCSC,k}))^{\beta})}, \tag{17}$$

$$p_{CNDPA}(i) = \frac{\alpha + (k_{CN,i})^{\beta}}{\sum_{j=1}^{N_N} (\alpha + (k_{CN,j})^{\beta})}, \tag{18}$$

respectively, and for Eqs. 16–18, $\alpha = 0.01$ and $\beta = 3$. In the above models—CSDPD, CSDPA, CNDRC and CNDPA—consilience optimization is not considered. To illustrate the importance of consilience optimization, another four models are also used. The first consilience optimization model assumes to have a central governor ignoring distance influence. This is a global optimization model and is denoted as CSDGO. The second consilience optimization model focuses on decentralized self-organizing systems, and distance is also not considered. This can be viewed as a local optimization model and is denoted as CSDLO. Then, based on CSDGO and CSDLO, distance influence is introduced to get another two consilience optimization models, denoted as CSDGOD and CSDLOD, respectively. In CSDGOD and CSDLOD, because of distance influence, the function $f_{CS}(\theta_i, \theta_j)$ needs to be modified as following:

$$\overline{f_{CS}}(\theta_i, \theta_j) = \begin{cases} f_{CS}(\theta_i, \theta_j)\left(\dfrac{d_{Max} - d_{i,j}}{(1-\delta)d_{Max}}\right)^{\varepsilon}, & d_{i,j} > \delta d_{Max} \\ f_{CS}(\theta_i, \theta_j), & d_{i,j} \le \delta d_{Max} \end{cases}, \quad (19)$$

where d_{Max} is the maximal connection length between nodes, and $0 \le \delta \le 1$ and $\varepsilon > 0$ are model parameters. Equation 19 implies that, if the connection length between two nodes is smaller than the threshold δd_{Max}, then distance has no influence on the original function $f_{CS}(\theta_i, \theta_j)$. Basically, a larger δ value means a less significant influence of distance. Above the threshold, distance influence becomes more significant as the ε value increases—in this study, $\delta = 0.1$ and $\varepsilon = 2$. To illustrate the differences in the outputs of the eight models, Fig. 3 gives eight relatively simple network systems.

To numerically understand the difference in the eight models, Table 1 gives some mean results of 100 runs of each model, where $N_{\mathrm{N}} = 100$ and $N_{\mathrm{E}} = 400$, CNDCC stands for CND-based clustering coefficient, Asso for assortativity in Newman (2002), ASPL for average shortest path length, CSD for consilience degree, NCSC for neighborhood consilience coefficient, and GCSC for global consilience coefficient. The degree distributions associated with Table 1 are plotted in Fig. 4. From Table 1 and Fig. 4, one may make the following observations. (1) In terms of CNDCC, Asso, or ASPL, CSDPD is similar to CNDRC, and CSDPA is similar to CNDPA. Since CNDCC, Asso, and ASPL are three basic CND-based network properties used to assess topology, we may conclude that topologies generated by CSDPD are similar to those of CNDRC, and CSDPA produces scale-free topologies as CNDPA does. The degree distributions in Fig. 4 also confirm the topology similarity between CNDRC and CSDPD, and between CNDPA and CSDPA. Therefore, topology-oriented properties can hardly distinguish CSDPD/CSDPA from CNDRC/CNDPA. (2) Regarding CSD, NCSC, or GCSC, one can clearly see that CSDPD/CSDPA is totally different from CNDRC/CNDPA, despite of their similarity in topology. This demonstrates that consilience-oriented properties may enable us to understand network systems from a new angle, which is completely missed by topology-oriented properties. (3) Comparing the details of CNDPA and CSDPA, one may echo the finding in Fig. 3, that is, CNDPA develops a scale-free pattern in topology faster than CSDPA. Usually, a more scale-free network has a shorter ASPL (hub nodes are more efficient to reach other nodes) and a larger maximal CND (given $N_{\mathrm{N}} = 100$, in CNDPA, some nodes have the theoretical maximal CND of 99, but in CSDPA, the maximal CND in all tests is less than 70). This is understandable, because, due to conflictive states between nodes, it takes much more time to develop a large CSD for Eq. 17 than to get a large CND for Eq. 18. (4) When comparing the four consilience optimization models (CSDGO, CSDGOD, CSDLO, and CSDLOD) with the four non-consilience-optimization models (CNDRC, CNDPA, CSDPD, and CSDPA), it is clear that, in terms of either topology-oriented properties or consilience-oriented properties, optimization models are rather different from non-optimization models. This implies that consilience optimization is a brand-new network problem, because neither CND-based models (such as CNDRC and CNDPA) nor CSD-based models (such as CSDPD and CSDPA) that borrow the techniques of CND-based models can effectively address the consilience optimization issue. Therefore, it demands innovative methods such as CSDGO, CSDGOD, CSDLO, and CSDLOD. (5) In Fig. 4, the four consilience optimization models have Poisson CND distributions, but it is worth further study to see whether consilience optimization models could have scale-free CND distributions.

4.2 Modeling Co-evolutionary Network Systems

Applying consilience degree (CSD) to study dynamic network systems is crucial to understanding and exploring the full potential of consilience theory. Here we demonstrate that CSD is an inherent property of dynamical network systems. As discussed in Sect. 3.4, many natural and social-ecological systems are co-evolutionary systems, where each node usually keeps changing its activity state and rewiring its connections according to its neighboring environment. Therefore, a fundamental question about the application potential of consilience theory is: Can CSD be used to model such real-world co-evolutionary network systems? To answer this question, we designed a CSD-based co-evolutionary network model where both node activity states and connections between nodes keep co-evolving under two highly realistic rules, that is, the selfish rule and the following-others rule. Basically, in many co-evolutionary, natural and social-ecological systems, these two major rules govern every node to change activity state

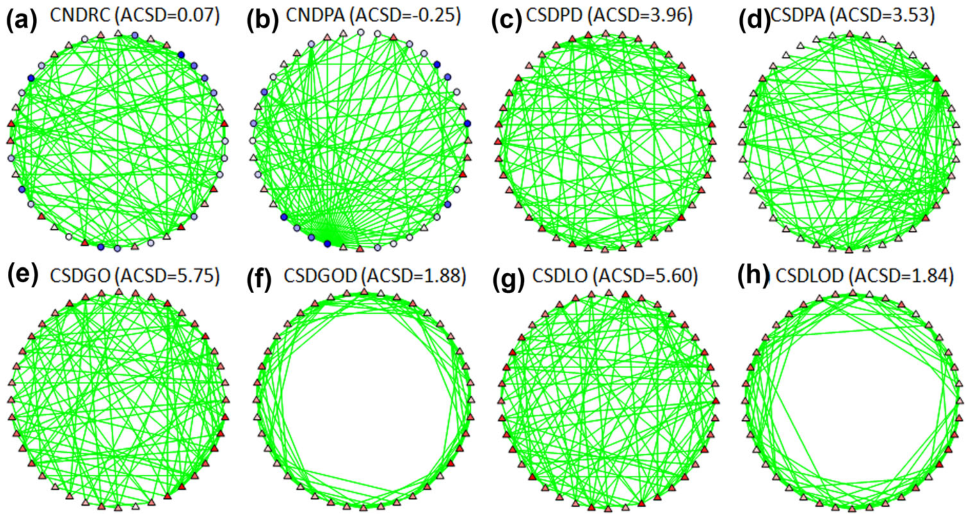


Fig. 3 Examples of networks generated by eight different models, where $N_{\mathrm{N}} = 40$ and $N_{\mathrm{E}} = 120$, the activity state of a node is randomly generated and then fixed, ACSD stands for average consilience degree, a triangle node means it has a positive CSD while a circle node has a negative one, and the color of node indicates the value of CSD (a deeper red means a larger positive value, while a deeper blue a larger absolute value of negative CSD). From this figure, one can see that: (1) For both CNDRC and CNDPA, the number of triangle nodes is similar to that of circle nodes, and the color of their nodes implies their CSDs are all around 0; (2) For both CSDPD and CSDPA, most nodes are a triangle with nearly red color, which means large positive CSD; (3) the topology of CSDPD is similar to that of CNDRC, and CSDPA is similar to CNDPA, which implies that network consilience cannot be determined solely by network topology; (4) Although CSDPA and CNDPA have the same values for α and β to calculate the connection probability, it seems that CNDPA develops a scale-free pattern in topology much faster than CSDPA; (5) In terms of ACSD, consilience optimization models CSDGO and CSDLO are significantly better than other models; (6) Once distance influence is introduced in CSDGOD and CSDLOD, ACSD decreases and so does the number of long connections, but optimization design still guarantees that all nodes have positive CSD; (7) The ACSD of global optimization models (CSDGO and CSDGOD) is always larger than the associated local optimization models (CSDLO and CSDLOD)

Table 1 Experimental results of different network models

	Topology-oriented properties			Consilience-oriented properties		
	CNDCC	Asso	ASPL	CSD	NCSC	GCSC
CNDRC	0.3071	0.0026	2.4256	− 0.0298	− 0.0031	− 0.0003
CNDPA	0.5224	0.3971	1.9296	− 0.0251	− 0.0034	− 0.0003
CSDPD	0.3515	0.0015	2.5778	5.7533	0.6329	0.0581
CSDPA	0.5975	0.3105	2.2817	4.8227	0.4881	0.0487
CSDGO	0.8109	− 0.0570	7.2882	7.9152	0.8663	0.0800
CSDGOD	0.6565	− 0.0424	3.5546	7.1922	0.7783	0.0726
CSDLO	0.7760	− 0.0130	6.9096	7.8713	0.8693	0.0795
CSDLOD	0.6057	− 0.0126	3.1937	6.8548	0.7514	0.0692

CNDCC stands for CND-based clustering coefficient, Asso for assortativity in Newman (2002), ASPL for average shortest path length, CSD for consilience degree, NCSC for neighborhood consilience coefficient, and GCSC for global consilience coefficient

and connections (Ball 2012). These rules can be well described based on the concept of CSD. Under the selfish rule, a node is more likely to change its activity state according to the states of supportive neighboring nodes, and it is also more likely to rewire a connection from a disturbing neighboring node to a supportive node. Under the following-others rule, all neighboring nodes are classified into two sets, supportive set and disturbing set. The node is more likely to change its activity state according to the set that has more nodes, and the node is also more likely to rewire a connection from the smaller set to a node that is connected to the larger set but currently not connected to the node. Figure 5 illustrates the basic ideas of the selfish rule and the following-others rule.

Now we give a mathematic description of the proposed CSD-based co-evolutionary network model. Suppose at

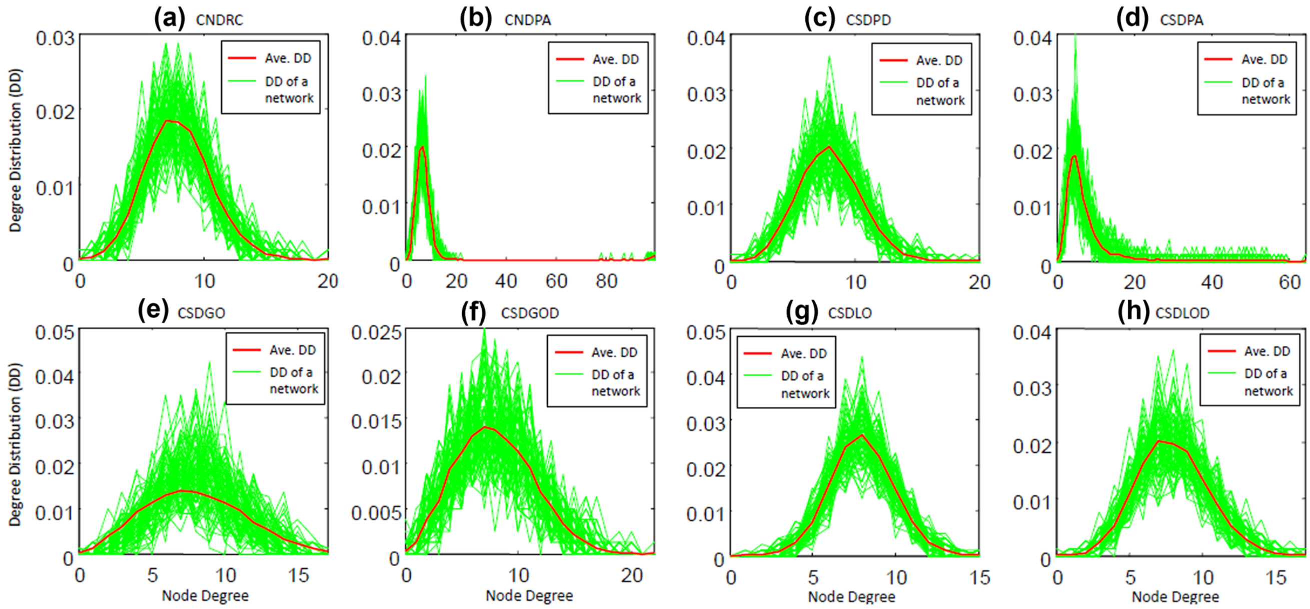


Fig. 4 Connection degree (CND) distributions associated with Table 1

time instant $t = 0$, we have an initial network system, where node activity states are distributed randomly within the range of $[0\ 2\pi]$, and connections between nodes are initialized according to the rule reported in Watts and Strogatz (1998). Basically, the initial network system is a random network without consilience design. Since the core of system dynamics is to self-adjust node activity states and connections, we focus the mathematical description on these two behaviors of a node. In this study, $f_{CS}(\theta_i, \theta_j)$ is set as $\cos(\theta_i\text{-}\theta_j)$.

Suppose at time instant $t \geq 0$, node i has $N_{SN,i}(t)$ neighboring nodes that are supportive (the set of such supportive neighboring nodes is denoted as $\Omega_{SN,i}(t)$), and $N_{DN,i}(t)$ neighboring nodes that are disturbing (the set of such disturbing neighboring nodes is denoted as $\Omega_{DN,i}(t)$).

If $N_{SN,i}(t) > 0$ and node i is adjusting its activity state $\theta_i(t)$ under the selfish rule at time instant t, then at the next time instant $t + 1$, its activity state will be

$$\theta_i(t+1) = \theta_i(t) + s_\theta \times \left(\frac{\sum_{j \in \Omega_{SN,i}(t)} \theta_j(t)}{N_{SN,i}(t)} - \theta_i(t) \right), \quad (20)$$

where s_θ is the speed of adjusting state. From Eq. 20, one can see that the state of node i is changing towards the mean value of all states of set $\Omega_{SN,i}(t)$.

If $N_{SN,i}(t) > 0$, $N_{DN,i}(t) > 0$, and node i is adjusting its connections under the selfish rule at time instant t, then it will randomly disconnect from a node in set $\Omega_{DN,i}(t)$ (assume node j is chosen), and then rewire the connection to a supportive node that is linked to set $\Omega_{SN,i}(t)$ but not to node i at time instant t (assume node k is chosen). After this adjustment, we have

$$\Omega_{SN,i}(t+1) = \Omega_{SN,i}(t) + \{k\}, N_{SN,i}(t+1) = N_{SN,i}(t) + 1, \quad (21)$$

$$\Omega_{DN,i}(t+1) = \Omega_{DN,i}(t) - \{j\}, N_{DN,i}(t+1) = N_{DN,i}(t) - 1. \quad (22)$$

If node i is adjusting its activity state $\theta_i(t)$ under the following-others rule at time instant t, then at the next time instant $t + 1$, its activity state will be

$$\theta_i(t+1) = \begin{cases} \theta_i(t) + s_\theta \times \left(\frac{\sum_{j \in \Omega_{SN,i}(t)} \theta_j(t)}{N_{SN,i}(t)} - \theta_i(t) \right), & N_{SN,i}(t) > N_{DN,i}(t) \\ \theta_i(t) + s_\theta \times \left(\frac{\sum_{j \in \Omega_{DN,i}(t)} \theta_j(t)}{N_{DN,i}(t)} - \theta_i(t) \right), & N_{DN,i}(t) > N_{SN,i}(t) \end{cases}. \quad (23)$$

From Eq. 23, one can see that node i will change its state to follow most of its neighboring nodes, no matter whether such majority neighboring nodes are currently supportive or disturbing to node i.

Suppose node i is adjusting its connections under the following-others rule at time instant t. If $N_{SN,i}(t) \geq N_{DN,i}(t) > 0$, then the connections of node i are changed in the same way as under the selfish rule according to Eqs. 21 and 22. If $0 < N_{SN,i}(t) < N_{DN,i}(t)$, then node i will randomly disconnect from a node in set $\Omega_{SN,i}(t)$ (assume node j is chosen), and then rewire the connection to a node that is supportively linked to set $\Omega_{DN,i}(t)$ but not to node i at time instant t (assume node k is chosen). After this adjustment, we have

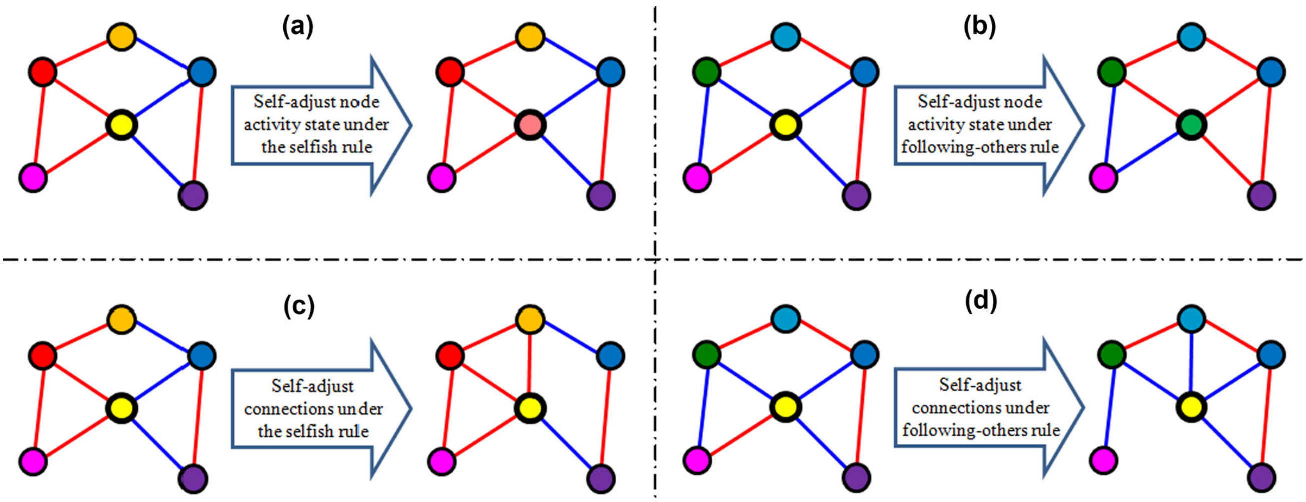


Fig. 5 Changing node activity state and connections under two co-evolutionary rules—selfish rule and following-others rule. **a** Self-adjust node activity state under the selfish rule; **b** self-adjust node activity state under the following-others rule; **c** self-adjust connections under the selfish rule; **d** self-adjust connections under the following-others rule. Suppose consilience function is set as $f_{CS}(\theta_i, \theta_j) = \cos(\theta_i - \theta_j)$. The similarity in node colors represents the similarity in node activity states. Red/Blue link represents positive/negative effect between nodes because of their similar/different states. The node with the bold boundary in the center of each network is the node that is currently adjusting its state/connections. In (**a**), the red and pink neighboring nodes are supportive of the central node because of their color similarity. Therefore, under the selfish rule, the central node changes its own state even more similar to those of the red and pink neighboring nodes, so that its own CSD will be increased. In (**b**), most neighboring nodes of the central node have cool colors. Therefore, under the following-others rule, the central node changes its own state from warm color to cool color in order to get more supportive effects from its neighboring nodes. In (**c**), under the selfish rule, the central node disconnects a negative neighboring node (the blue node), and rewires the connection to a supportive neighbor of its own supportive neighboring nodes. In this way, it stands a good chance to increase its CSD. In (**d**), the central node has only 1 supportive neighbor, but 3 negative neighbors. Therefore, under the following-others rule, the central node disconnects the only supportive neighboring node (the pink node), and rewires the connection to a supportive neighbor of its own negative neighboring nodes. After this adjustment, its CSD decreases for the moment, but if the central node adjusts its state according to the selfish rule in the future, its CSD will increase significantly

$$\Omega_{DN,i}(t+1) = \Omega_{DN,i}(t) + \{k\}, N_{DN,i}(t+1) = N_{DN,i}(t) + 1, \quad (24)$$

$$\Omega_{SN,i}(t+1) = \Omega_{SN,i}(t) - \{j\}, N_{SN,i}(t+1) = N_{SN,i}(t) - 1. \quad (25)$$

At each time instant of the co-evolutionary process, the percentage of nodes that are randomly chosen to change activity states is P_{CAS}, and the percentage of nodes that are randomly chosen to rewire connections is P_{RWC}. Given that node i is chosen to evolve at time instant t, the probability of applying the selfish rule is calculated as follows:

$$P_{SR,i}(t) = \alpha(i) + (1 - \alpha(i)) \times \frac{N_{SN,i}(t)}{N_{SN,i}(t) + N_{DN,i}(t)}, \quad (26)$$

where $0 \le \alpha(i) \le 1$ is a coefficient that indicates how selfish node i is. A larger $\alpha(i)$ means more selfish. In this study, for the sake of simplicity, we set $\alpha(i) = 0.3$ for all nodes. Based on $P_{SR,i}(t)$, the probability of applying the following-others rule is simply

$$P_{FO,i}(t) = 1 - P_{SR,i}(t). \quad (27)$$

With the co-evolutionary dynamics defined by Eqs. 20–27, an initial network system without consilience design will gradually develop good network consilience during the co-evolutionary process, as illustrated in Fig. 6. Given the generality of co-evolution in reality, we therefore argue that CSD is an inherent attribute rather than an artificial concept, which underpins the fundamental importance of CSD to the study of real-world complex network systems such as social-ecological systems.

5 Conclusion

To study the performance of a system against disturbances, many important concepts have been developed, such as "robustness" in systems science, and "vulnerability," "resilience," and "adaptive capacity" in social-ecological systems. A question is: Have these existing concepts fully described the performance of a system against disturbances? In the practice of real-world disaster and risk management, the consensus of wills and coordination of activities in a society often play a crucial role, which however can hardly be reflected or captured by existing concepts. This article proposes a new, fundamental, general network property—consilience degree (CSD), which is especially used to evaluate how well a system has

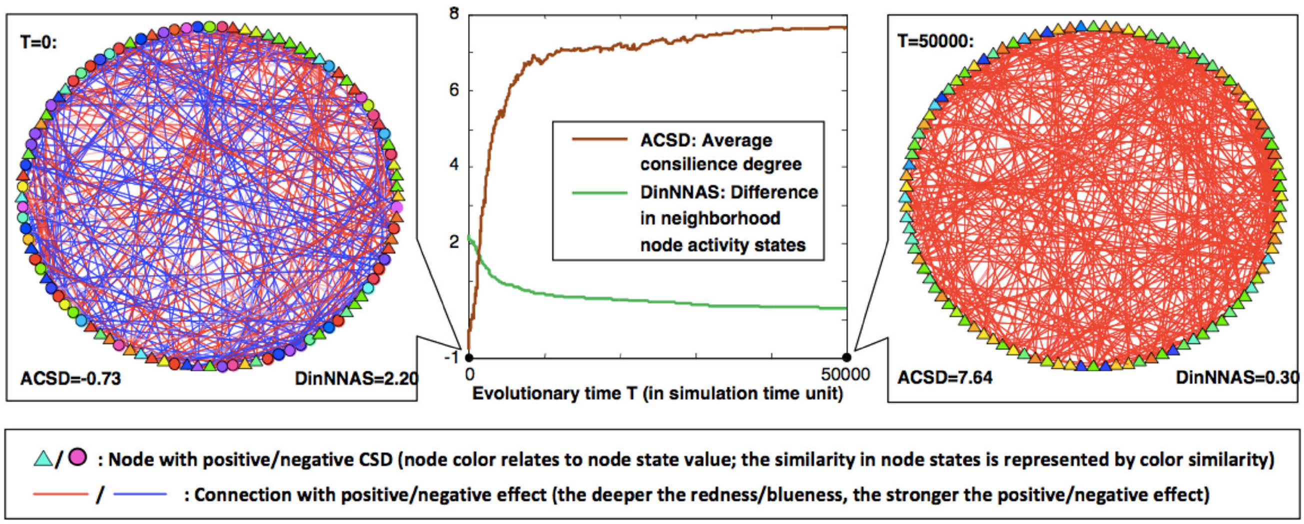


Fig. 6 Co-evolutionary mechanism causes average consilience degree (ACSD) to increase. The system has 100 nodes and 400 edges, and thus the potential maximal ACSD is 8 in theory. In the co-evolutionary process, each node can autonomously change its activity state and connections according to its neighboring environment. The change is guided by a selfish rule and a following-others rule. Under the selfish rule, a node changes its activity state and connections by referring to those neighbors that have similar activity states to its own. Under the following-others rule, a node changes its activity state and connections by referring to the majority neighbors that share similar activity states (maybe rather different from its own). At the initial time (T = 0), the system is randomly set up, and therefore the ACSD is almost 0. During the co-evolutionary process, the difference in neighborhood node activity states (DinNNAS) decreases gradually, and as a result, the ACSD increases to 7.64, nearly the potential maximal ACSD, at the end of the simulation. Since the co-evolutionary mechanism in the simulation is widely observed in reality, one might then conclude that a larger ACSD is generally desirable to real-world network systems

integrated and coordinated resources, in order to serve a specific systemic goal such as dealing with disturbances. Actually, CSD can be viewed as a generalized node connection degree (CND). In this article, with the basic idea of CSD, a set of new network properties and models are developed that form a new theoretical framework to study complex systems. As a static network property, CSD also exhibits great potential to study dynamical network systems. In particular, a CSD-based co-evolutionary network model is developed in this article that proves that CSD is an inherent attribute rather than an artificial concept.

Our theoretical analyses and simulation results prove that CSD-based network properties and models are rather different from CND-based network properties and models, and they open a new window to deepen our understanding of many real-world complex systems such as social-ecological systems (SES). For instance, a society that has a consensus of wills and practices a coordination of activities between individuals for the sake of disaster prevention, mitigation, and relief is often observed to be less vulnerable to disasters (Shi et al. 2014). In the stage of disaster prevention, whether and to what extent individuals compete for or share resources will make a difference in the preparedness level against disasters. In the stage of disaster mitigation and relief, whether and to what extent individuals loot or help each other may amplify or reduce the impact of disasters. Although concepts such as vulnerability, resilience, and adaptive capability are fundamentally important to study SES, they largely fail to address these issues. Hopefully, CSD can be used to quantify and improve the performance of SES against disasters (Shi et al. 2014). In coping with global environmental change, multiple stakeholders in SES keep changing their attitudes, behaviors, interactions, and relationships. Co-evolutionary consilience models may thus help to make SES healthier and more sustainable. Therefore, it is worth further efforts to apply the new CSD theories and models in real-world case studies of SES.

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